ME 5321: Advanced Classical Thermodynamics
Homework 4

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Problem 1

Problem Statement

Nitrogen gas is being withdrawn from a 0.15 m^3 cylinder at a rate of 10 mol/min. The cylinder initially contains the gas at a pressure of 100 bar and 170 K. The cylinder is well insulated and there is negligible heat transfer between the cylinder walls and the gas.

(a) How many moles of gas will be in the cylinder at any time?

(b) What are the temperature and pressure of the gas in the cylinder after 50 minutes?

Determine (a) and (b) for the following cases:

(i) Assuming that nitrogen is an ideal gas.

(ii) Using the pressure-enthalpy diagram of nitrogen.

(iii) Assuming that nitrogen follows the van der Waals equation of state.

Solution

The energy equation for an open system is

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t}\iiint \rho(u + \frac{1}{2}V^2 + gz)\,d\mathcal{V} + \iint \rho(h + \frac{1}{2}V^2 + gz)(\mathbf{V}\cdot\hat{\mathbf{n}})\,dA \tag{1}$$

Neglecting any change in kinetic and potential energy, and noting that no heat transfer is taking place and no work is done on the control volume,

$$\frac{\partial}{\partial t}\iiint \rho u\,d\mathcal{V} + \iint \rho h(\mathbf{V}\cdot\hat{\mathbf{n}})\,dA = 0 \tag{2}$$

Application of this equation to the control volume for the tank, assuming properties are uniform inside the tank, yields

$$\frac{d}{dt}(m\,u) + \dot{m}_{\text{out}}h = 0 \tag{3}$$

Which becomes

$$m\frac{du}{dt} + u\frac{dm}{dt} = -\dot{m}_{\text{out}}h \tag{4}$$

Now, conservation of mass must be applied.

$$\frac{\partial}{\partial t}\iiint \rho \, d\mathcal{V} + \iint \rho(\mathbf{V}\cdot\hat{\mathbf{n}})\, dA = 0 \tag{5}$$

Again, assuming properties are uniform in the cylinder,

$$\frac{dm}{dt} = -\dot{m}_{\text{out}} \tag{6}$$

Rearranging and integrating,

$$\int_{m_0}^{m(t)} dm = -\int_0^t \dot{m}_{\text{out}}\, dt \tag{7}$$

Because the flow rate out is constant,

$$\boxed{m(t) = m_0 - \dot{m}_{\text{out}} t} \tag{8}$$

Inserting (8) into (4),

$$(m_0 - \dot{m}_{\text{out}} t)\frac{du}{dt} - \dot{m}_{\text{out}} u = -\dot{m}_{\text{out}} h \tag{9}$$

Assuming a calorically perfect gas,

$$\int_{T_0}^{T(t)} \frac{1}{T}\, dT = \int_0^t \frac{1-\gamma}{\frac{m_0}{\dot{m}_{\text{out}}} - t}\, dt \tag{10}$$

Evaluating the integral yields

$$\boxed{T(t) = T_0\left(1 - \frac{\dot{m}_{\text{out}}}{m_0}\, t\right)^{\gamma-1}} \tag{11}$$

Now, the pressure as a function of time can be found using the ideal gas law again.

$$\boxed{p(t) = \frac{m(t)}{\mathcal{V}} RT(t) = \frac{RT_0}{\mathcal{V}}\left(m_0 - \dot{m}_{\text{out}} t\right)\left(1 - \frac{\dot{m}_{\text{out}}}{m_0}\, t\right)^{\gamma-1}} \tag{12}$$

Evaluating these expressions at 50 minutes yields $p = 6.86546$ MPa and $T = 152.68$ K.

Using the pressure enthalpy diagram requires the following steps:

- Find the initial enthalpy using the initial pressure and temperature (red dot on the chart).
- Find the enthalpy as a function of time using the fact that the process can be approximated as isentropic (blue line).
- Use conservation of mass to determine what the density will be at $t = 50$ min and use this information to find the final temperature and pressure.

The density at 50 minutes is 151.503 kg/m^3 which is equivalent to 9.458 lb/ft^3. The final state is marked by the green circle on the chart.

The mass as a function of time is the same as before:

$$m(t) = m_0 - \dot{m}t$$

The mass in the tank after 50 minutes is 22.7255 kg. This can easily be converted to the number of moles in the tank using the typical unit conversion.

The pressure and temperature after 50 minutes are 3.75 MPa and 126.2 K, respectively.

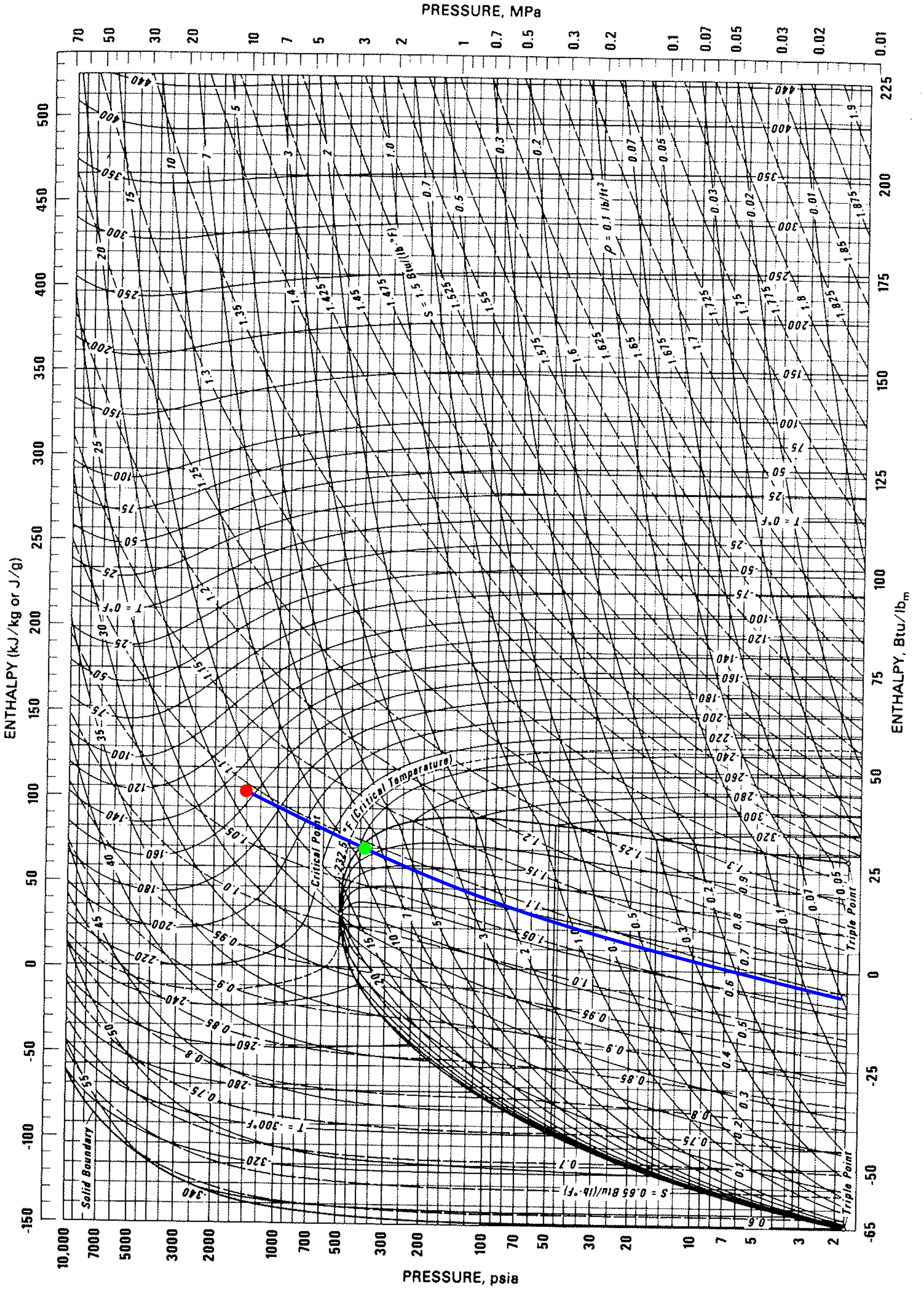


Figure 2.4-3

Pressure–enthalpy diagram for nitrogen (*Source:* Cryogenics Division, NBS-IBS, Boulder, Colorado, Chart 2028, April 1, 1977. Used with permission.)

Table 1: Comparison between three methods.

	ideal gas	diagram	van der Waals
T [K]	152.68	126.2	152.68
P [MPa]	6.865	3.75	4.689

Now, for the van der Waals EOS, the assumption of constant specific heats is made again. Therefore,

$$du = c_v \, dT$$
$$dh = c_p \, dT$$

and

$$u = c_v T \tag{14a}$$
$$h = c_p T \tag{14b}$$

Going back to the energy and continuity equations:

$$m(t) = m_0 - \dot{m}_{\text{out}} t$$

and

$$m \frac{du}{dt} + u \frac{dm}{dt} = -\dot{m}_{\text{out}} h$$

Using the same manipulations as before, we arrive at the same equation for temperature.

$$\boxed{T(t) = T_0 \left(1 - \frac{\dot{m}_{\text{out}}}{m_0} t\right)^{\gamma-1}} \tag{15}$$

The equation for pressure as a function of time will be different because we are using the van der Waals EOS.

$$\left(p(t) + a\frac{m(t)^2}{\mathcal{V}^2}\right)(\mathcal{V} - m(t)b) = m(t)RT(t) \tag{16}$$

Rearranging,

$$\boxed{p(t) = \frac{m(t)RT(t)}{\mathcal{V} - bm(t)} - a\left(\frac{m(t)}{\mathcal{V}}\right)^2} \tag{17}$$

Inserting the previously derived relations and evaluating the expressions for temperature and pressure at 50 minutes yields $p = 4.689$ MPa and $T = 152.68$ K. A table comparing the three results is provided below.

As long as the diagram was used correctly, it should be the most accurate. The results in the above table show that the van der Waals equation of state yields a more realistic value of pressure than the ideal gas equation of state. This is to be expected since the van der Waals EOS includes more information than the ideal gas EOS.

Problem 2

Problem Statement

Ten grams of liquid water at 95°C are contained in an insulated container. The initial volume is 1×10^{-5} m^3. A pin holding a frictionless piston in place breaks and the volume available to the water increases to 1×10^{-3} m^3. During the expansion, some of the water evaporates, but no heat is transfered to the cylinder. Find the temperature, pressure and amounts of vapor, liquid and solid water after the expansion.

Solution

The initial specific volume is 0.001 m^3/kg. The final specific volume is 0.1 m^3/kg. Assuming the liquid is saturated, because the final specific volume is less than the specific volume for a saturated vapor, the mixture must be a combination of vapor and liquid. Also, because the temperature and pressure do not change during a phase change, the final temperature and pressure are 95°C and 84.609 kPa, respectively. The quality can be found using $v_f = 0.001040$ m^3/kg and $v_g = 1.9808$ m^3/kg. The quality of the mixture is 0.04998.

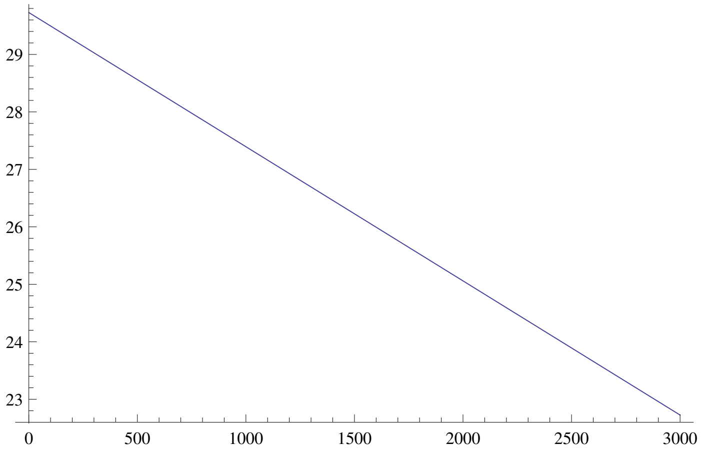
Code listing

Homework 4 problem 1a

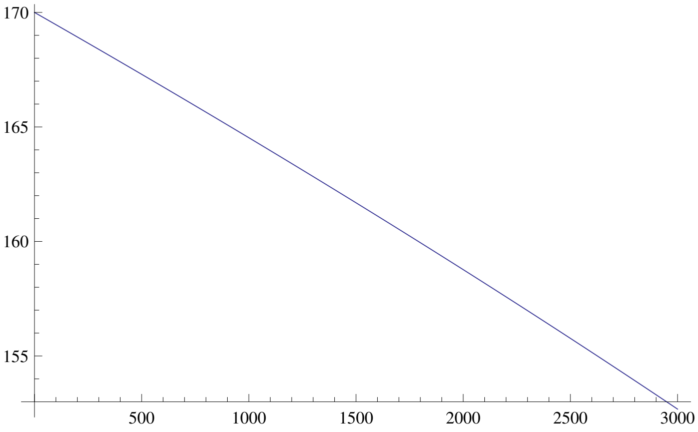
```
ClearAll["Global`*"]

T0 = 170;

p0 = 100 * 100 000;

V = 0.15;

mdot = 10 * 14.0067 / 1000 / 60;

R = 296.8;

γ = 1.4;

m0 = p0 * V / (R T0);

ρ = m[3000] / V
151.503
```

Ideal gas

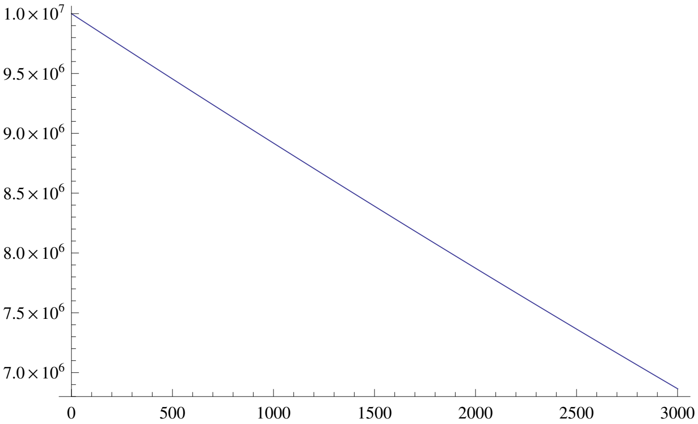
```
m[t_] := m0 - mdot * t

T[t_] := T0 (1 - mdot / m0 t) ^ (γ - 1)

p[t_] := m[t] / V R T[t]

Plot[m[t], {t, 0, 3000}]
```



```
Plot[T[t], {t, 0, 3000}]
```



```
Plot[p[t], {t, 0, 3001}]
```



```
m[3000]
```

```
22.7255
```

```
p[3000]
```

6.86546×10^{6}

```
T[3000]
```

```
152.68
```

Homework 4 problem 1c

```
In[188]:= ClearAll["Global`*"]

In[189]:= T0 = 170;

In[190]:= p0 = 100 * 100 000;

In[191]:= V = 0.15;

In[192]:= mdot = 10 * 14.0067 / 1000 / 60;

In[193]:= R = 296.8;

In[194]:= γ = 1.4;

In[195]:= m0 = p0 * V / (R T0);
```

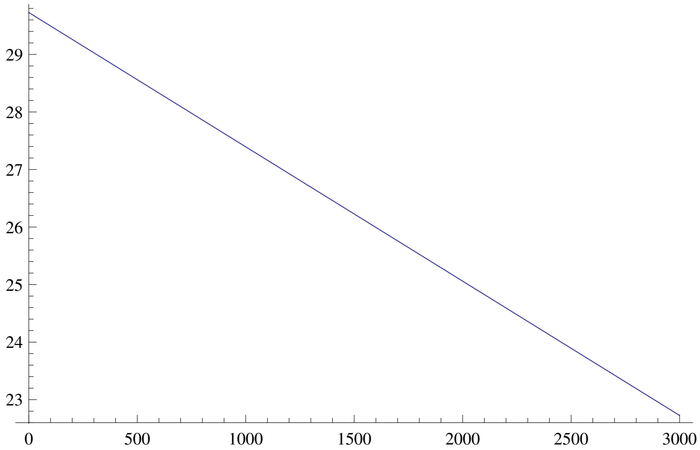
---

van der Waals

```
In[196]:= Tc = 126;

In[197]:= Pc = 34 × 10^5;

In[198]:= a = 27 R^2 Tc^2 / (64 Pc)

Out[198]= 173.53

In[199]:= b = R Tc / (8 Pc)

Out[199]= 0.00137488

In[200]:= m[t_] := m0 - mdot * t

In[201]:= T[t_] := T0 (1 - mdot / m0 t) ^ (γ - 1)

In[202]:= p[t_] := m[t] R T[t] / (V - m[t] b) - a (m[t] / V) ^2

In[203]:= p[0]

Out[203]= 6.92925 × 10^6
```

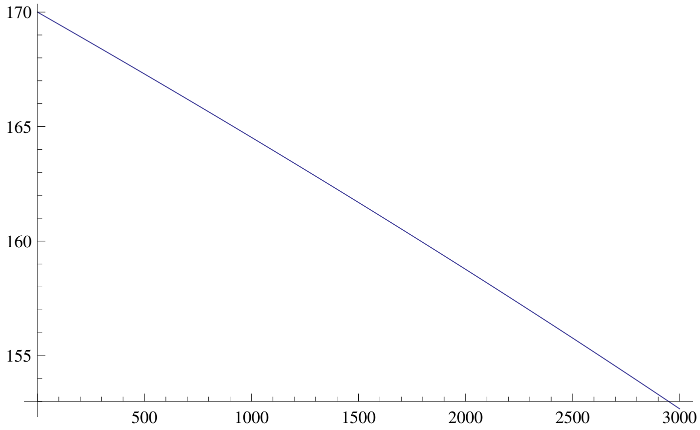
In[204]:= `Plot[m[t], {t, 0, 3000}]`

Out[204]=



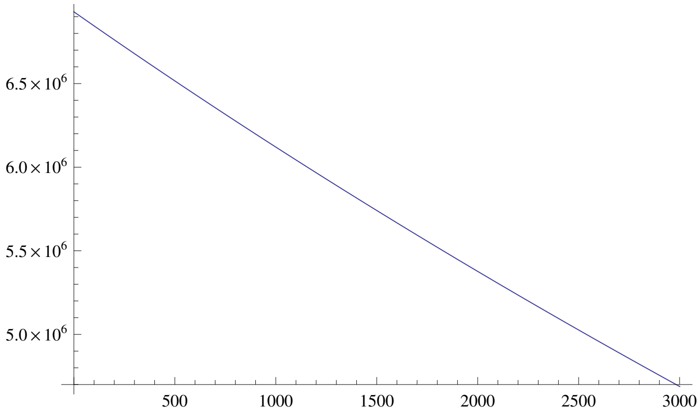
In[205]:= `Plot[T[t], {t, 0, 3000}]`

Out[205]=



In[206]:= `Plot[p[t], {t, 0, 3001}]`

Out[206]=



In[207]:= `m[3000]`

Out[207]= `22.7255`

```
In[208]:= p[3000]
```

Out[208]= 4.68871×10^{6}

```
In[209]:= T[3000]

Out[209]= 152.68
```