AE 5381: Boundary Layers
Homework 2

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Problem 2.1

Problem Statement

Assume that the velocity profile over a flat plate may be approximated as

$$\frac{u}{U_e} = \tanh\left[2.65\left(\frac{y}{\delta}\right)\right] \tag{1}$$

Calculate $\delta(x)$ and $C_f(x)$.

Solution

The dimensionless velocity profile given in (1) is shown in Figure 1.

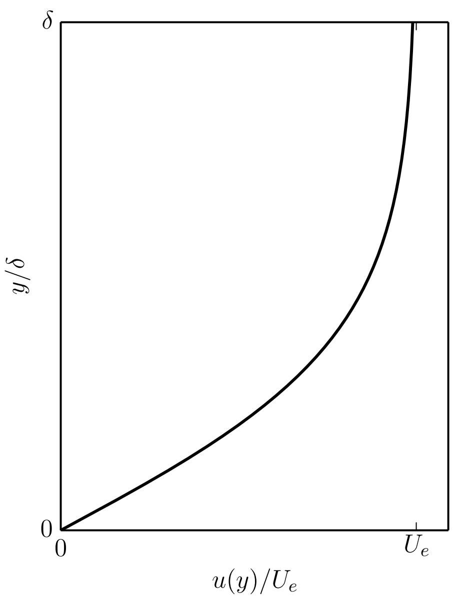


Figure 1: Dimensionless velocity profile.

Application of the integral momentum equation to a control volume in the boundary layer of length dx and height H, where $H > \delta$, yields a differential equation that relates the inviscid velocity (U_e), the displacement thickness, momentum thickness, and skin friction coefficient. This equation is

$$\frac{d\theta}{dx} + \frac{1}{U_e}\frac{dU_e}{dx}(2\theta + \delta^*) - \frac{v_w}{U_e} = \frac{C_f}{2} \tag{2}$$

where the momentum thickness is

$$\theta \equiv \int_0^\delta \left(1 - \frac{u}{U_e}\right)\frac{u}{U_e}\,dy \tag{3}$$

the displacement thickness is

$$\delta^* \equiv \int_0^\delta \left(1 - \frac{u}{U_e}\right)dy \tag{4}$$

and the skin friction coefficient is

$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U_e^2} \tag{5}$$

Assuming the inviscid solution is available (i.e., U_e is known), we have one equation, (2), and three unknowns: θ, δ^*, and C_f. However, if the velocity profile is known (or assumed), then (3), (4), and (5) are all known.

The inviscid solution for a flat plate corresponds to a uniform stream with a constant velocity in the x-direction. The flat plate corresponds to one streamline. Thus, U_e is a constant. Also, the bottom wall is solid so $v_w = 0$. Inserting these into (2) yields

$$\frac{d\theta}{dx} = \frac{C_f}{2} \tag{6}$$

Letting $a = 2.65$ and Inserting the given velocity profile into (3),

$$\theta = \int_0^\delta \left(1 - \tanh\left[\frac{ay}{\delta}\right]\right)\tanh\left[\frac{ay}{\delta}\right]\,dy$$

Using Mathematica[1] to evaluate the integral yields

$$\theta = \frac{\delta\ln(\cosh(a - \ln(\cosh(a - \tanh(a)))))}{a} = 0.3739\,\delta$$

The shear stress on the wall is given by

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$\frac{\partial u}{\partial y} = \frac{2.65\,U_e\,\mathrm{sech}^2\left(\frac{2.65y}{\delta}\right)}{\delta}$$

So,

$$\tau_w = \frac{2.65\,\mu\,U_e}{\delta}$$

[1] Code is listed in the appendix.

Thus,

$$C_f = \frac{5.3\mu}{\delta\,\rho\,U_e} \tag{7}$$

Now, (6) becomes

$$\delta\,\frac{d\delta}{dx} = \frac{14.1735\,\mu}{\rho\,U_e} \tag{8}$$

Cross multiplying and integrating,

$$\int \delta\,d\delta = \int \frac{14.1735\,\mu}{\rho\,U_e}\,dx$$

which yields

$$\delta(x) = 5.3242\,\sqrt{\frac{\mu\,x}{\rho\,U_e}} + C$$

Since this problem involves a flat plate, the boundary layer thickness at $x = 0$ is zero. Therefore,

$$\delta(x) = 5.3242\,\sqrt{\frac{\mu\,x}{\rho\,U_e}}$$

This equation can be recast in terms of Reynolds number as follows:

$$\mathrm{Re}_x = \frac{\rho\,U_e\,x}{\mu}$$

$$\boxed{\delta(x) = 5.3242\,x\,\mathrm{Re}_x^{-1/2}} \tag{9}$$

Now, $C_f(x)$ can be found using (7) and (9).

$$\boxed{C_f(x) = 0.9955\,\mathrm{Re}_x^{-1/2}} \tag{10}$$

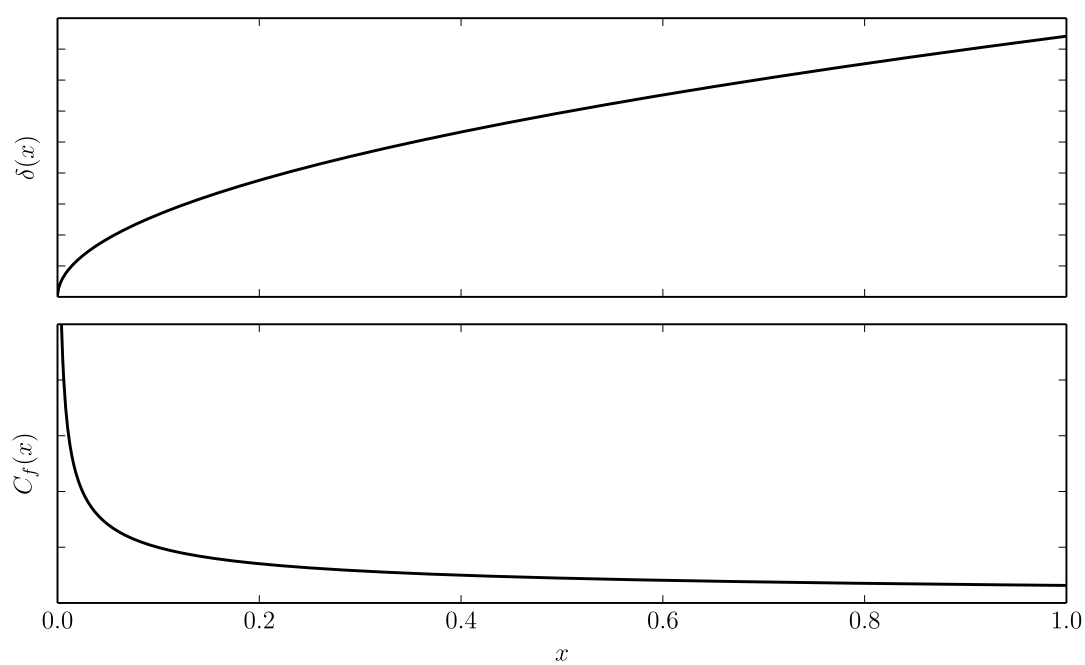


Figure 2: Boundary layer thickness and skin friction coefficient as a function of x.

Problem 2.3

Problem Statement

Consider a flat plate in a fluid medium that is at rest, except for a line *sink* located on the surface at $x = L$, as shown in the following sketch. Suppose the inviscid velocity field produced by the sink is $v_r = -C(L/r)$, where r is the radial distance from the sink to any point in the flow. Determine the momentum thickness distribution $\theta(x)$.

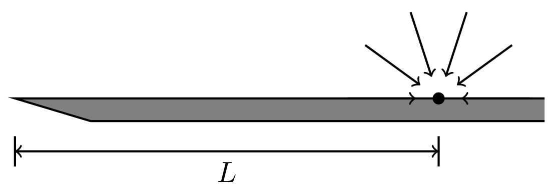


Figure 3: Problem schematic.

Solution

Approach:

- Determine stream function.
- Use coordinate transformation to determine $\mathbf{V}(x, y)$.
- Two options:
  (a) Use Pohlhausen's assumed velocity profile and integrate to find $\theta(x)$.
  (b) Use Thwaites-Walz Method to solve for $\theta(x)$ directly.

The x- and y-components of velocity can be written in terms of the stream and potential functions as follows:

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \tag{11}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{12}$$

In polar coordinates,

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

So,

$$\frac{\partial \psi}{\partial \theta} = -CL$$

Partially integrating,

$$\psi(r,\theta) = \int -CL\,d\theta + f(r)$$

$$\psi(r,\theta) = -CL\,\theta + f(r)$$

Differentiating both sides with respect to r,

$$\frac{\partial \psi}{\partial r} = f'(r) = v_\theta = 0$$

So,

$$\psi(r,\theta) = -CL\,\theta$$

The coordinate transformation from polar to cartesian is

$$r = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Using this, the stream function is

$$\psi(x,y) = -CL\tan^{-1}\left(\frac{y}{x-L}\right)$$

Now, using (11) and (12),

$$u = \frac{L-x}{(L-x)^2+y^2} \tag{13}$$

$$v = \frac{-y}{(L-x)^2+y^2} \tag{14}$$

The streamline that represents the flat plate ($y = 0$) yields U_e:

$$U_e(x) = \frac{1}{(L-x)} \tag{15}$$

This inviscid solution is the input for the boundary layer solution. First, the Pohlhausen Method will be applied. The assumed velocity profile is

$$\frac{u}{U_e} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2 + d\left(\frac{y}{\delta}\right)^3 + e\left(\frac{y}{\delta}\right)^4 \tag{16}$$

where

$$a = 0$$

$$b = 2 + \frac{\lambda}{6}$$

$$c = -\frac{\lambda}{2}$$

$$d = -2 + \frac{\lambda}{2}$$

$$e = 1 - \frac{\lambda}{6}$$

The variable λ is the *Pohlhausen pressure gradient parameter* which is given by

$$\lambda = \frac{\delta^2}{\nu}\frac{dU_e}{dx}$$

For this problem,

$$\lambda = \frac{\delta^2}{\nu(L-x)^2}$$

Thus,

$$a = 0 \tag{17a}$$

$$b = \frac{\delta^2}{6\nu(L-x)^2} + 2 \tag{17b}$$

$$c = -\frac{\delta^2}{2\nu(L-x)^2} \tag{17c}$$

$$d = \frac{\delta^2}{2\nu(L-x)^2} - 2 \tag{17d}$$

$$e = 1 - \frac{\delta^2}{6\nu(L-x)^2} \tag{17e}$$

Now, turning to (2),

$$\frac{d\theta}{dx} + \frac{1}{U_e}\frac{dU_e}{dx}(2\theta + \delta^*) - \frac{v_w}{U_e} = \frac{C_f}{2} \tag{2}$$

Since the wall is solid, $v_w = 0$. Taking the derivative of (15) with respect to x yields

$$\frac{dU_e}{dx} = \frac{1}{(L-x)^2}$$

So,

$$\frac{1}{U_e}\frac{dU_e}{dx} = \frac{1}{L-x}$$

Inserting (17) into (16),

$$u(x,y) = \frac{U_e\, y\left(2\,\delta^3 + y^3 - 2\delta y^2\right)}{\delta^4} - \frac{U_e\, y\,(y-\delta)^3}{6\,\delta^2\,\nu(L-x)^2} \tag{18}$$

The momentum thickness is given by (3) which is repeated here.

$$\theta \equiv \int_0^\delta \left(1 - \frac{u}{U_e}\right)\frac{u}{U_e}\,dy \tag{3}$$

Evaluating this equation yields

$$\theta(x) = \frac{\delta}{45360}\left(\frac{-5\delta^4 - 48\delta^2\nu(L-x)^2}{\nu^2(L-x)^4} + 5328\right)$$

However, $\delta(x)$ is still unknown. Equation (2) must be solved for $\delta(x)$. Inserting all that is known yields

$$\frac{-5\delta^5\rho^2(2LU_egc - 2U_ecx + 1) - 3\delta^3\mu\rho(L-x)^2(16U_e(L-x) + 79) + 12132\delta\mu^2(L-x)^4}{22680\mu^2U_e(L-x)^6} = \frac{\frac{2\mu}{\delta\rho} + \frac{\delta}{6(L-x)^2}}{U_e}$$

This equation is a quintic polynomial in δ and as such does not have a closed form solution. Next, the Thwaites-Walz Method will be used. Multiplying (2) by $U_e\theta/\nu$ yields

$$\frac{\tau_w\theta}{\mu U_e} = \frac{U_e\theta}{\nu}\frac{d\theta}{dx} + \frac{\theta^2 dU_e/dx}{\nu}(H+2) \tag{19}$$

where H, named the *shape factor*, is a nondimensional function of the profile shape only. The term on the left hand side is also a function of the profile shape alone and is called the *shear correlation function*, denoted by S. Letting $\Lambda = (\theta^2/\nu)dU_e/dx$, (19) becomes

$$U_e\frac{d}{dx}\left[\frac{\Lambda}{dU_e/dx}\right] = 2\left\{S(\Lambda) - \Lambda(H(\Lambda)+2)\right\} = F(\Lambda) \tag{20}$$

where $F(\Lambda)$ comes from a curve fit from a large amount of experimental data available.

$$F(\Lambda) = 0.45 - 6.0\Lambda$$

Using this, (20) can be integrated which yields

$$\theta^2(x) = \frac{0.45\nu}{U_e^6(x)}\int_0^x U_e^5(x')\,dx + \theta^2(0)\left[\frac{U_e(0)}{U_e(x)}\right]^6 \tag{21}$$

For a sharp-nosed body, $\theta(0) = 0$ because the boundary layer thickness is zero at $x = 0$. However, $U_e(x)$ is already known from (15). Inserting all of this into (21) and evaluating yields

$$\theta(x) = 0.67082\sqrt{\frac{\mu x(L-x)}{\rho}}$$

A plot of the momentum thickness is shown below.

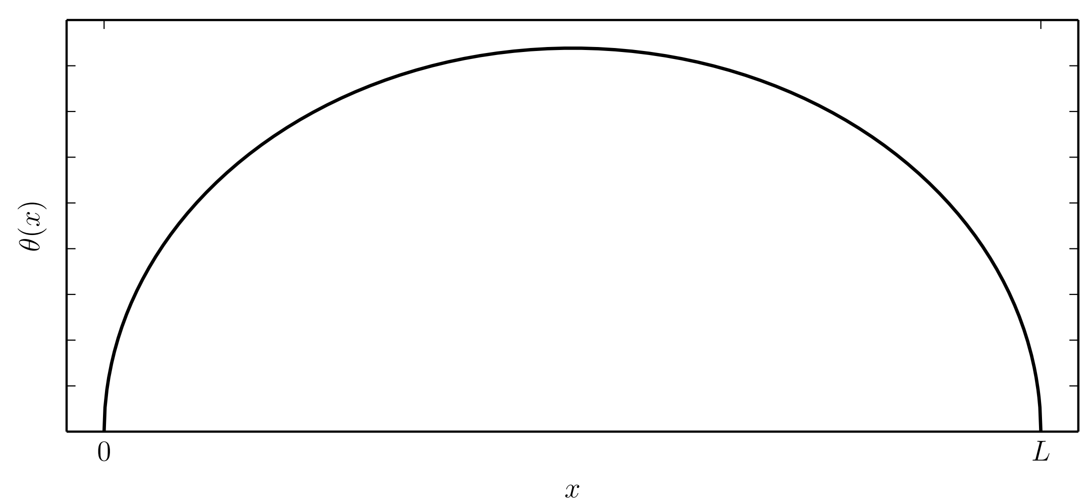


Figure 4: Plot of momentum thickness over flat plate with sink at L.

Problem 2.7

Problem Statement

Air at 1.0 atm and room temperature flows over a plate at a rate of 3.0 m/s. Determine the mass flow (per unit width) of external stream fluid that enters the boundary layer between the leading edge and a station 50 cm from the leading edge.

Solution

The mass flow of external stream fluid that enters the boundary layer between the leading edge and a station 50 cm from the leading edge is equivalent to determining the mass flow rate at 50 cm. The justification for the preceding statement is based on the fact that the boundary layer deflects the streamlines upward by an amount equivalent to the displacement thickness, δ^*, and $\delta^* < \delta$. Therefore, this problem is equivalent to

$$\dot{m} = \int_0^\delta \rho\, u(y)\, dy$$

The problem is that $u(y)$ is not known. This is where Pohlhausen's method comes in. Pohlhausen's velocity profile will be used and $\delta(x)$ will be found along the way. The velocity profile is given by

$$\frac{u}{U_e} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2 + d\left(\frac{y}{\delta}\right)^3 + e\left(\frac{y}{\delta}\right)^4 \tag{22}$$

For this case, the Pohlhausen pressure gradient parameter is zero. That is,

$$\lambda = \frac{\delta^2}{\nu}\frac{dU_e}{dx} = 0 \;\because\; U_e = \text{const}$$

So, (22) becomes

$$u(y) = U_e\left(\frac{y^4}{\delta^4} - \frac{2y^3}{\delta^3} + \frac{2y}{\delta}\right) \tag{23}$$

However, the boundary layer thickness, $\delta(x)$ is unknown. Turning to the adapted momentum equation,

$$\frac{d\theta}{dx} + \frac{1}{U_e}\frac{dU_e}{dx}(2\theta + \delta^*) - \frac{v_w}{U_e} = \frac{C_f}{2} \tag{2}$$

which becomes

$$\frac{d\theta}{dx} = \frac{C_f}{2} \tag{24}$$

where θ is the momentum thickness which is given by (3).

$$\theta = \int_0^\delta \left(1 - \frac{u}{U_e}\right)\frac{u}{U_e}\, dy = \int_0^\delta \left(1 - \frac{y^4}{\delta^4} + \frac{2y^3}{\delta^3} - \frac{2y}{\delta}\right)\left(\frac{y^4}{\delta^4} - \frac{2y^3}{\delta^3} + \frac{2y}{\delta}\right) dy$$

Evaluating the integral yields

$$\theta(x) = \frac{37}{315}\delta(x)$$

Inserting this into (24) and remembering the definition for C_f,

$$\frac{37}{315}\frac{d\delta}{dx} = \frac{\tau_w}{\rho U_e^2}$$

and

$$\tau_w = \mu\frac{\partial u}{\partial y} = \mu U_e\left[\frac{2}{\delta} + \frac{4y^3}{\delta^4} - \frac{6y^2}{\delta^3}\right]_{y=0} = \frac{2\mu U_e}{\delta}$$

Now,

$$\delta\frac{d\delta}{dx} = \frac{630}{37}\frac{\mu}{\rho U_e}$$

Using separation of variables,

$$\delta(x) = \sqrt{\frac{1260}{37}\frac{\mu x}{\rho U_e}} = 5.8356\,x\,\mathrm{Re}_x^{-1/2}$$

Using this, the mass flow (per unit width) of external stream fluid in the boundary layer at station x is

$$\boxed{\dot{m} = 4.0849\sqrt{\mu\,\rho\,U_e\,x} = 4.0849\,\mu\,\mathrm{Re}_x^{1/2}}$$

For this problem,

$$\begin{aligned} U_e &= 3.0 \text{ m/s} \\ \rho &= 1.225 \text{ kg/m}^3 \\ \mu &= 1.789 \times 10^5 \text{ N·s/m}^2 \end{aligned}$$

Thus

$$\boxed{\dot{m} = 0.023421 \text{ kg/(s·m)}}$$

Problem 2.9

Problem Statement

What is $C_f(x)$ in the vicinity of the stagnation point for air at standard temperature and pressure (STP) flowing at 5.0 ft/s over a circular cylinder with a 1.0-in. diameter? Use the Thwaites-Walz method.

Solution

Approach:

- Determine inviscid solution for flow over a cylinder using potential flow.
- Apply Thwaites-Walz method to determine C_f.

The potential flow solution for flow over a cylinder involves the superposition of a doublet and a uniform stream. A doublet is actually a superposition of a source sink pair that are of strength m and $-m$, respectively that are separated by a distance a as $a \to 0$. The complex potential for flow around a cylinder is

$$F(z) = \phi + i\psi = V_\infty \left(z + \frac{R^2}{z} \right)$$

where $z = x + iy$ or in polar coordinates, $z = re^{i\theta}$. The stream function is the imaginary part of the complex potential. That is

$$\psi(r, \theta) = \mathrm{Im}[F(z)]$$

Now, the velocity components can be determined by differentiating the complex potential.

$$w(z) = \frac{dF}{dz} = u - iv = (u_r - i\, u_\theta)e^{i\theta}$$

Evaluating the derivative (in polar coordinates) yields the complex velocity

$$w(z) = \left(1 - \frac{R^2}{r^2} e^{-2i\theta} \right)$$

Using Euler's formula and separating real and imaginary parts yields the velocity compoents:

$$u_r = V_\infty \cos\theta \left(1 - \frac{R^2}{r^2} \right) \tag{25}$$

$$u_\theta = -V_\infty \sin\theta \left(1 + \frac{R^2}{r^2} \right) \tag{26}$$

We are interested in the velocity at the surface of the cylinder which can be determined by evaluating (26) at $r = R$. Doing so yields

$$U_e = -2V \sin\theta$$

However, $0 \leqslant \theta \leqslant \pi$ measured counter clockwise from the positive x-axis. We need the component of velocity along the surface of the cylinder. A simple mapping can be used to accomplish this. Let

$$\theta = \frac{s}{R} - \pi$$

where s represents the distance along the surface of the cylinder from the stagnation point on the left. Now, U_e becomes

$$U_e(s) = 2V_\infty \sin\left(\frac{s}{R} \right)$$

The derivative of U_e wrt s is needed for the Thwaites-Walz method.

$$\frac{dU_e}{ds} = \frac{2V_\infty}{R} \cos\left(\frac{s}{R} \right)$$

If $s = 0$,

$$\left(\frac{dU_e}{ds} \right)_0 = \frac{2V_\infty}{R}$$

This result answers a question asked in class about how we determine dU_e/ds. The momentum thickness must now be determined,

$$\theta_t^2(s) = \frac{0.45\nu}{U_e^6(s)} \int_0^s U_e^5(s')\, ds' + \theta_t^2(0) \left[\frac{U_e(0)}{U_e(s)} \right]^6$$

where θ_t has been used to denote the momentum thickness because the symbol θ has already been used. For this problem, $U_e(0) = 0$ because of the stagnation point on the leading edge. Evaluating the integral yields

$$\theta_t(s) = \frac{0.08385}{V_\infty^3 \sin\left(\frac{s}{R}\right)} \sqrt{\nu \left(32RV_\infty^5 \left(-\frac{1}{5}\cos^5\left(\frac{s}{R}\right) + \frac{2}{3}\cos^3\left(\frac{s}{R}\right) - \cos\left(\frac{s}{R}\right)\right) + \frac{256R}{15}V_\infty^5\right)}$$

Now, $\Lambda(s)$ can be calculated.

$$\Lambda(s) = \frac{\theta_t^2}{\nu}\frac{dU_e}{ds}$$

Now, the shear correlation function needs to be determined. Using the fit to experimental data given on page 46 of the book,

$$S(\Lambda) = 0.22 + 1.57\Lambda - 1.80\Lambda^2$$

Then, the desired result is

$$C_f = \frac{2\mu}{\rho U_e \theta_t} S(\Lambda) \tag{27}$$

All values in (27) are known. Evaluating (27) yields a rather large symbolic expression. The result can be seen in the appendix. The skin friction coefficient is infinite at the stagnation point, as shown in the below figure.

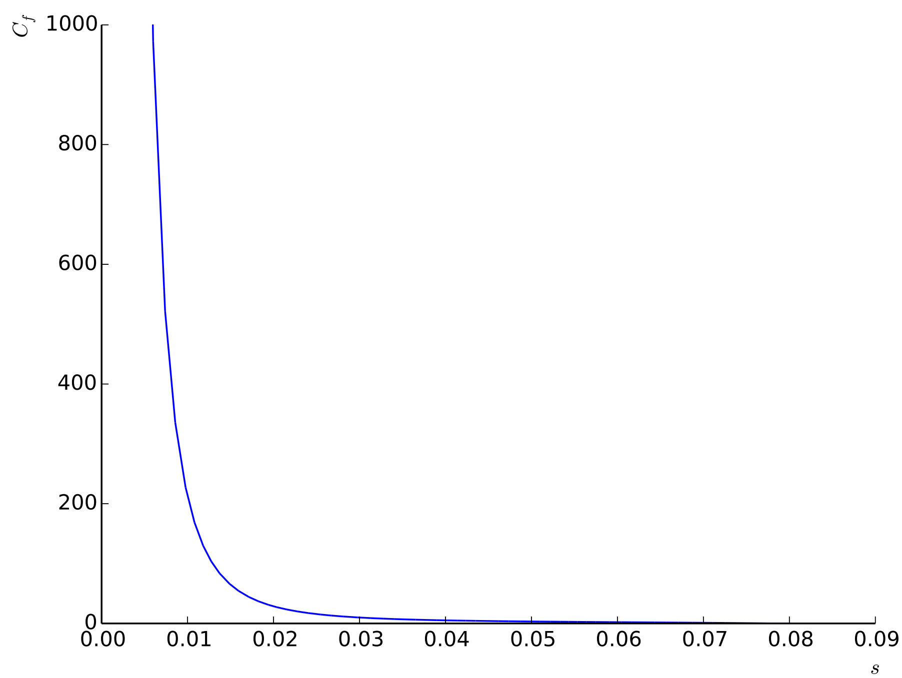


Figure 5: Plot of skin friction coefficient vs s.

Problem 2.16

Problem Statement

For a plate heated over its entire length, how is the average value of the film coefficient up to a station x related to the local value at that same station?

Solution

For this problem $h(x)$ must be determined, and then h_{avg} will be computed by integration. The ratio between the two film coefficients will then be compared. The assumed velocity profile is given by

$$\frac{u}{U_e} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

And the assumed temperature profile is

$$\frac{T - T_w}{T_e - T_w} = \frac{3}{2}\frac{y}{\delta_T} - \frac{1}{2}\left(\frac{y}{\delta_T}\right)^3$$

where δ_T is the thickness of the thermal boundary layer. Because this is a flat plate, $dU_e/dx = 0$. Thus, (2) simplifies to

$$\frac{d\theta}{dx} = \frac{1}{2}C_f = \frac{\tau_w}{\rho U_e^2} = \frac{\nu}{U_e^2}\frac{\partial u}{\partial y}\bigg|_{y=0} \tag{28}$$

The momentum thickness is determined in the same manner as before

$$\theta(x) = \int_0^\delta \left(1 - \frac{u}{U_e}\right)\frac{u}{U_e}\,dy$$

Evaluating the integral yields

$$\theta(x) = \frac{39}{280}\delta(x)$$

Determining the velocity gradient in the y direction, inserting all of the previously mentioned information into (28), and simplifying yields

$$\frac{39}{280}\frac{d\delta}{dx} = \frac{3\nu}{2U_e\delta}$$

Solving this ODE yields

$$\delta(x) = \sqrt{\frac{280}{13}\frac{\nu x}{U_e}}$$

The relationship between the thermal and velocity boundary layers is given by

$$\frac{\delta_T}{\delta} = \frac{1}{1.026\,\text{Pr}^{1/3}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

In this case, the entire plate is heated so $x_0 = 0$ and the above equation becomes

$$\frac{\delta_T}{\delta} = \frac{1}{1.026\,\text{Pr}^{1/3}}$$

where the Prandtl number is given by

$$\mathrm{Pr} \equiv \frac{\mu c_p}{k}$$

Now, the temperature profile can be written as

$$T(x) = \left[\frac{3}{2}\frac{y}{\delta_T} - \frac{1}{2}\left(\frac{y}{\delta_T}\right)^3\right](T_e - T_w) + T_w$$

Now, the heat transfer can be written as

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \equiv h(T_w - T_e)$$

Therefore,

$$h(x) = \frac{-k}{T_w - T_e}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

The temperature gradient can be determined directly from the temperature profile.

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{0.331613(T_e - T_w)\sqrt[3]{\frac{\mu c_p}{k}}}{\sqrt{\frac{\nu x}{U_e}}}$$

Now,

$$h(x) = \frac{0.331613k\sqrt[3]{\frac{\mu c_p}{k}}}{\sqrt{\frac{\nu x}{U_e}}}$$

The average film coefficient over some length of the plate, denoted by ℓ, is given by

$$h_{\mathrm{avg}} = \frac{1}{\ell}\int_0^{\ell} h(x)\,dx$$

Now,

$$\frac{h(x)}{h_{\mathrm{avg}}} = \frac{1}{2}\sqrt{\frac{\ell}{x}}$$

This equation is plotted in Figure 6. This figure shows that h_{avg} is reasonably close to the actual value over a lot of the plate, except for the beginning. If the length used to compute the average film coefficient is equal to x, then the relation becomes

$$\boxed{\frac{h(x)}{h_{\mathrm{avg}}} = \frac{1}{2}}$$

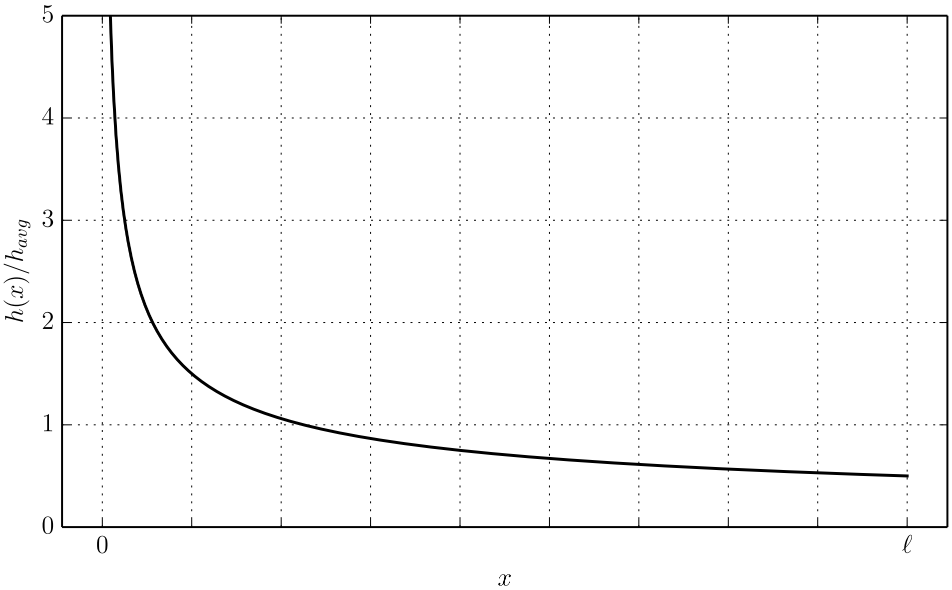


Figure 6: Comparison between local and average (taken over length ℓ) film coefficients vs x.

Problem 2.18

Problem Statement

Hydrogen at 500°C and 15 atm flows over a plate at 2 m/s. The first 2 cm of the plate are maintained at that same temperature, the next 3 cm are at 600°C, and the rest of the plate is at 650°C. What is the heat transfer rate 6 cm from the leading edge?

Solution

The integral energy equation is

$$\frac{d}{dx}\left[\int_0^H (T_e - T)\, u\, dy\right] + \frac{\nu}{c_p}\int_0^H \left(\frac{\partial u}{\partial y}\right)^2 dy - v_w(T_e - T_w) = \frac{-k(\partial T/\partial y)_w}{\rho c_p} \tag{29}$$

Because this equation is linear in T, the principle of superposition can be applied. That is, given two or more solutions to a linear differential equation, their sum is also a solution. Therefore, the total heat flow at a location x is equal to the sum of the individual solutions.

$$q_w = \sum_i h(x, \xi_i)(\Delta T_{wi})$$

The film coefficient is given by (2.58) from the text.

$$h(x, \xi_i) = 0.332\,\mathrm{Pr}^{1/3}\frac{k}{x}\left(\frac{\rho U_e x}{\mu}\right)^{1/2}\left[1 - \left(\frac{\xi_i}{x}\right)^{3/4}\right]^{-1/3}$$

Performing the sum yields

$$q_w = \frac{k\sqrt[3]{\text{Pr}}\left(\frac{0.332\Delta T_1}{\sqrt[3]{1-\left(\frac{\xi_1}{x}\right)^{3/4}}} + \frac{0.332\Delta T_2}{\sqrt[3]{1-\left(\frac{\xi_2}{x}\right)^{3/4}}}\right)\sqrt{\frac{\rho U_e x}{\mu}}}{x}$$

Using the tables at the back of the book to determine the properties of hydrogen yields

$$\boxed{q_w(6\text{ cm}) = 1.5643\text{ W/m}^2}$$

CAS Notebooks

Chapter 2 Problem 1

In[103]:= ClearAll["Global`*"]

In[104]:= u[y_] := Ue * Tanh[2.65 * y / δ]

In[105]:= Integrate[(1 - u[y] / Ue) * u[y] / Ue, {y, 0, δ}]

Out[105]= $\int_0^{\delta}\left(1 - \mathrm{Tanh}\left[\frac{2.65\,y}{\delta}\right]\right)\mathrm{Tanh}\left[\frac{2.65\,y}{\delta}\right]dy$

In[106]:= int1 = Integrate[Tanh[a * y / δ], y]

Out[106]= $\frac{\delta\,\mathrm{Log}\left[\mathrm{Cosh}\left[\frac{a\,y}{\delta}\right]\right]}{a}$

In[107]:= int2 = Integrate[Tanh[a * y / δ] ^ 2, y]

Out[107]= $y - \frac{\delta\,\mathrm{Tanh}\left[\frac{a\,y}{\delta}\right]}{a}$

In[108]:= θ = int1 /. y → δ - int1 /. y → 0 + int2 /. y → δ - int2 /. y → 0 // FullSimplify

Out[108]= $\frac{\delta\,\mathrm{Log}[\mathrm{Cosh}[a - \mathrm{Log}[\mathrm{Cosh}[a - \mathrm{Tanh}[a]]]]]}{a}$

In[109]:= θ = θ /. a → 2.65

Out[109]= $0.373936\,\delta$

In[110]:= dudy = D[u[y], y] // FullSimplify

Out[110]= $\frac{2.65\,\mathrm{Ue}\,\mathrm{Sech}\left[\frac{2.65\,y}{\delta}\right]^2}{\delta}$

In[111]:= τw = μ * dudy /. y → 0

Out[111]= $\frac{2.65\,\mathrm{Ue}\,\mu}{\delta}$

In[112]:= Cf = τw / (1 / 2 * ρ * Ue^2)

Out[112]= $\frac{5.3\,\mu}{\mathrm{Ue}\,\delta\,\rho}$

In[113]:= RHS = Cf / θ * δ^2

Out[113]= $\frac{14.1735\,\mu}{\mathrm{Ue}\,\rho}$

In[114]:= δ[x_] := Sqrt[2.0 * RHS * x]

```
In[115]:= Cfsymb = 5.3 * μ / (Sqrt[2.0 * RHS * x] * ρ * Ue) // FullSimplify
```

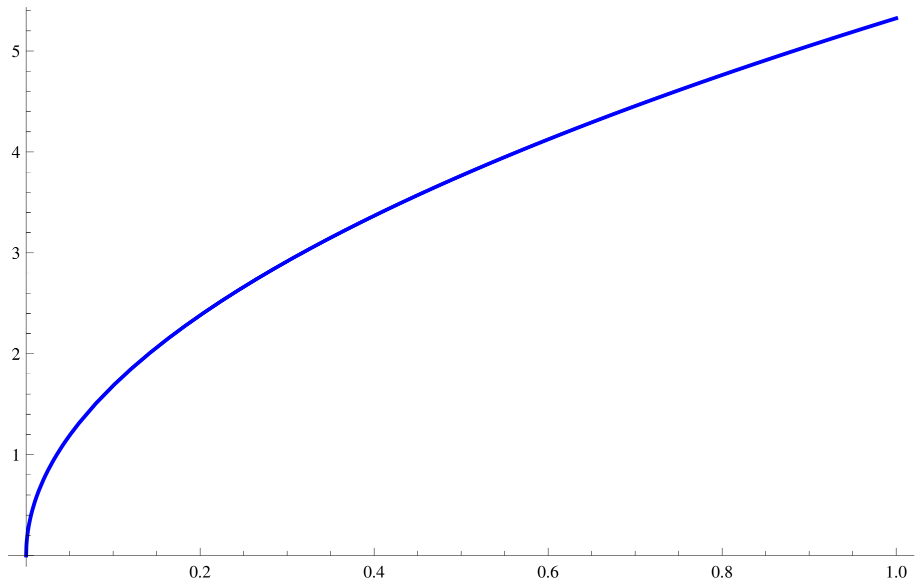
Out[115]= $$\frac{0.995455\sqrt{\frac{x\,\mu}{Ue\,\rho}}}{x}$$

```
In[116]:= Sqrt[2.0 * RHS * x]
```

Out[116]= $$5.3242\sqrt{\frac{x\,\mu}{Ue\,\rho}}$$

```
In[117]:= Plot[δ[x] /. μ → 1 /. Ue → 1 /. ρ → 1, {x, 0, 1}, PlotStyle → {Thick, Blue}]
```

Out[117]=



Chapter 2 Problem 3

In[42]:=
```
ClearAll["Global`*"]
```

Finding *u* and *v* using the transformation matrix

In[43]:=
```
β = {{Cos[θ], Sin[θ]}, {-Sin[θ], Cos[θ]}}
```

Out[43]= {{Cos[θ], Sin[θ]}, {-Sin[θ], Cos[θ]}}

In[44]:=
```
βinv = Inverse[β] // FullSimplify
```

Out[44]= {{Cos[θ], -Sin[θ]}, {Sin[θ], Cos[θ]}}

In[45]:=
```
Vpolar = {-1 / r, 0};
```

In[46]:=
```
Vcartesian = βinv.Vpolar /. θ → ArcTan[y, x] /. r → Sqrt[x^2 + y^2] // FullSimplify
```

Out[46]= $\left\{-\frac{y}{x^2+y^2}, -\frac{x}{x^2+y^2}\right\}$

Finding *u* and *v* by solving for the stream function and using x=r*cos(theta) ...

In[47]:=
```
v = -D[-ArcTan[y / (x - L)], x] // FullSimplify
```

Out[47]= $-\frac{y}{(L-x)^2+y^2}$

In[48]:=
```
u = D[-ArcTan[y / (x - L)], y] // FullSimplify
```

Out[48]= $\frac{L-x}{(L-x)^2+y^2}$

In[49]:=
```
Ue[x_] := u /. y → 0
```

In[50]:=
```
Ue[xx]
```

Out[50]= $\frac{1}{L-x}$

In[51]:=
```
dUedx = D[Ue[x], x]
```

Out[51]= $\frac{1}{(L-x)^2}$

In[52]:=
```
1 / Ue[x] * dUedx
```

Out[52]= $\frac{1}{L - x}$

Applying Pohlhausen’s method

In[53]:=
```
λ = δ^2 / ν * dUedx
```

Out[53]= $\frac{\delta^2}{(L - x)^2 \nu}$

In[54]:=
```
a = 0;
```

In[55]:=
```
b = 2 + λ / 6 // FullSimplify
```

Out[55]= $2 + \frac{\delta^2}{6 (L - x)^2 \nu}$

In[56]:=
```
c = -λ / 2
```

Out[56]= $-\frac{\delta^2}{2 (L - x)^2 \nu}$

In[57]:=
```
d = -2 + λ / 2 // FullSimplify
```

Out[57]= $-2 + \frac{\delta^2}{2 (L - x)^2 \nu}$

In[58]:=
```
e = 1 - λ / 6
```

Out[58]= $1 - \frac{\delta^2}{6 (L - x)^2 \nu}$

In[83]:=
```
uu = Ue[x] * (a + b * y / δ + c * (y / δ) ^2 + d * (y / δ) ^3 + e * (y / δ) ^4) // FullSimplify
```

Out[83]= $\frac{y \left(6 (L - x)^2 (y^3 - 2 y^2 \delta + 2 \delta^3) + \frac{\delta^2 (-y+\delta)^3 \rho}{\mu}\right)}{6 (L - x)^3 \delta^4}$

In[85]:=
```
θ = Integrate[(1 - uu / Ue[x]) * uu / Ue[x], {y, 0, δ}] // FullSimplify
```

Out[85]= $\frac{\delta \left(5328 + \frac{\delta^2 \rho (-48 (L-x)^2 \mu - 5 \delta^2 \rho)}{(L-x)^4 \mu^2}\right)}{45360}$

In[86]:=
```
D[θ, x] // FullSimplify
```

Out[86]= $\frac{\delta^3 \rho (-24 (L - x)^2 \mu - 5 \delta^2 \rho)}{11340 (L - x)^5 \mu^2}$

In[87]:=
```
δs = Integrate[(1 - uu / Ue[x]), {y, 0, δ}] // FullSimplify
```

Out[87]=
$$\frac{1}{120}\,\delta\left(36-\frac{\delta^2\,\rho}{(L-x)^2\,\mu}\right)$$

In[88]:=
```
τw = μ * D[uu, y] /. y → 0 // FullSimplify
```

Out[88]=
$$\frac{2\,\mu}{L\,\delta-x\,\delta}+\frac{\delta\,\rho}{6\,(L-x)^3}$$

In[94]:=
```
Cf = τw / (1 / 2 * ρ * Ue[x] ^2) // FullSimplify
```

Out[94]=
$$\frac{\delta}{3\,L-3\,x}+\frac{4\,(L-x)\,\mu}{\delta\,\rho}$$

In[90]:=
```
ν = μ / ρ;
```

In[92]:=
```
LHS = D[θ, x] + 1 / Ue[x] * dUedx * (2 * θ + δs) // FullSimplify
```

Out[92]=
$$\frac{4044\,(L-x)^4\,\delta\,\mu^2-95\,(L-x)^2\,\delta^3\,\mu\,\rho-5\,\delta^5\,\rho^2}{7560\,(L-x)^5\,\mu^2}$$

In[95]:=
```
RHS = 1 / 2 * Cf // FullSimplify
```

Out[95]=
$$\frac{\delta}{6\,L-6\,x}+\frac{2\,(L-x)\,\mu}{\delta\,\rho}$$

In[96]:=
```
soln = Solve[ LHS == RHS , δ]
```

Out[96]=

A very large output was generated. Here is a sample of it:

$$\Bigg\{\Bigg\{\delta\to-\sqrt{\Bigg(-\frac{19\,L^2\,\mu}{3\,\rho}+\frac{38\,L\,x\,\mu}{3\,\rho}-\frac{19\,x^2\,\mu}{3\,\rho}+\frac{10157\times 2^{\ll 1\gg}\,L^4\,\mu^2\,\rho}{3\,(\ll 1\gg)^{1/3}}-\frac{\ll 1\gg}{3\,\ll 1\gg}+\frac{\ll 1\gg}{\ll 1\gg^{\ll 1\gg}}-\frac{40628\times 2^{1/3}\,L\,x^3\,\mu^2\,\rho}{3\,(\ll 1\gg+\ll 9\gg+\ll 1\gg)^{1/3}}+\frac{10157\times 2^{1/3}\,x^4\,\mu^2\,\rho}{3\,(\ll 1\gg)^{1/3}}+\frac{\left(-23822350\,L^6\,\mu^3\,\rho^6+\ll 9\gg+\sqrt{4\,\ll 1\gg^3+\ll 1\gg^2}\right)^{1/3}}{15\times 2^{1/3}\,\rho^3}\Bigg)}\Bigg\},\ \ll 4\gg,\ \left\{\delta\to\sqrt{\ll 1\gg}\right\}\Bigg\}$$

Show Less | Show More | Show Full Output | Set Size Limit...

In[69]:=
```
(* FullSimplify[soln,{μ ∈Reals,L∈Reals,x∈Reals,Ue∈Reals,δ∈Reals,ρ∈Reals}] *)
```

Thwaites-Walz Method

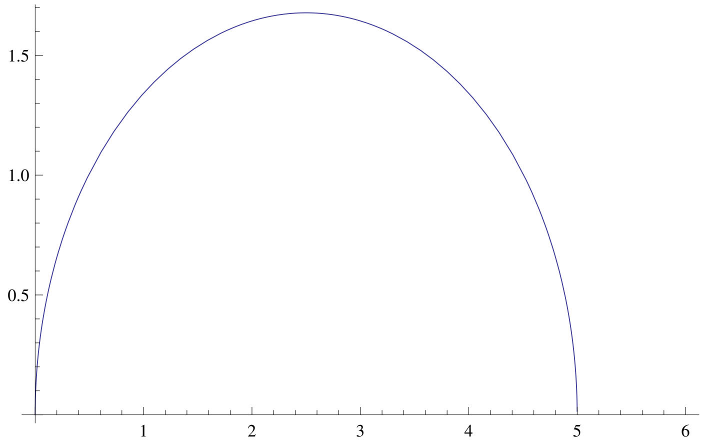
In[97]:=
```
Ue[x]
```

Out[97]=
$$\frac{1}{L-x}$$

In[108]:=
```
θ = Sqrt[0.45 * ν / Ue[x] ^ 6 * Integrate[Ue[xp] ^ 5, {xp, 0, x}]] // FullSimplify
```

Out[108]= $0.67082 \sqrt{\frac{(L - x)\, x\, \mu}{\rho}}$

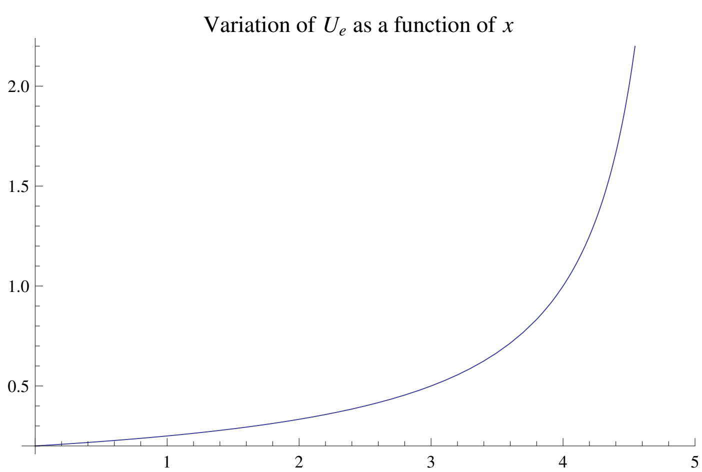
In[111]:=
```
Plot[θ /. ν → 1 /. L → L0 /. ρ → 1, {x, 0, 6}]
```

Out[111]=


Plotting

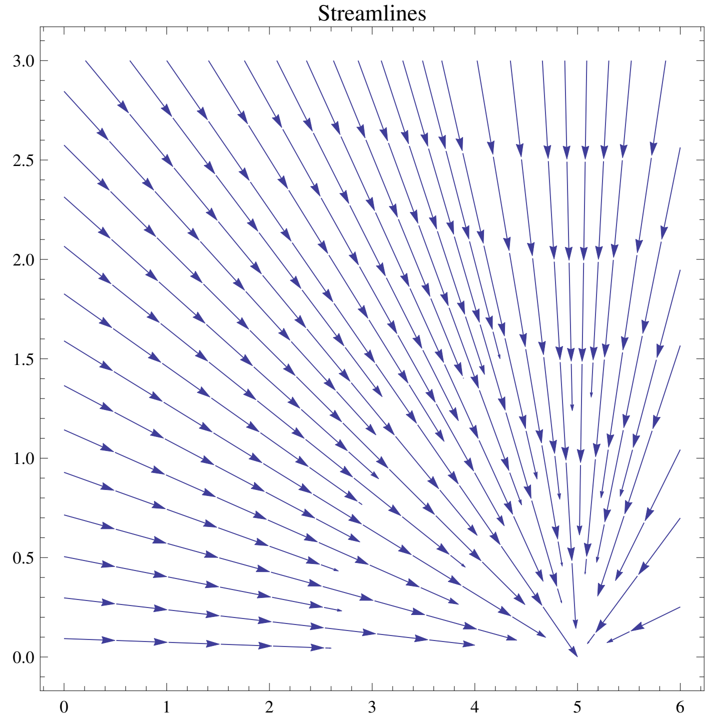
In[71]:=
```
L0 = 5;
```

In[72]:=
```
Plot[Ue[x] /. L → L0, {x, 0, L0 - 0.1},
  PlotLabel → "Variation of Ue as a function of x"]
```

Out[72]=


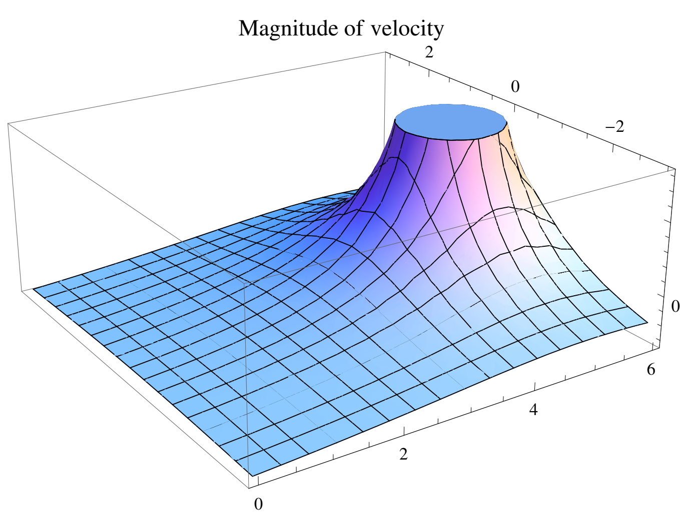
In[73]:=
```
StreamPlot[{u /. L → L0, v /. L → L0},
 {x, 0, L0 + 1}, {y, 0, 3}, PlotLabel → "Streamlines"]
```

Out[73]=



In[74]:=
```
Plot3D[Sqrt[u^2 + v^2] /. L → L0, {x, 0, L0 + 1},
 {y, -3, 3}, PlotLabel → "Magnitude of velocity"]
```

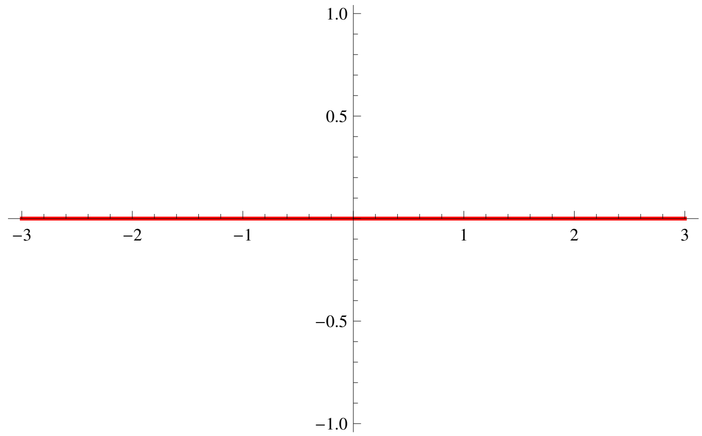
Out[74]=



In[75]:=
```
ψ[x_, y_] := -ArcTan[y / x]
```

In[76]:=
```
ψ0 = {0.0};
```

In[77]:=
```
streamline = Solve[ψ[x, y] == ψ0[[1]], y];
sl[x_] := y /. streamline[[1]];
mph = Plot[sl[x], {x, -3, 3}, PlotStyle → {Thick, Red}];
Print[mph];
```



AE_5381_Hw2

October 8, 2014

```
In [1]: from sympy import *

In [2]: import numpy as np

In [3]: init_printing()

In [4]: x, y, r, theta = symbols("x y r theta", real=True)

In [5]: V = Symbol("V", real=True, positive=True)

In [6]: R = Symbol("R", real=True, positive=True)

In [7]: z = Symbol("z")

In [8]: z_cart = x + I*y

In [9]: z_pol = r*exp(I*theta)
```

Complex Potential

```
In [10]: phi = V*(z + R**2/z)

In [11]: diff(phi, z)
```

Out[11]:

$$V \left(- \frac{R^{2}}{z^{2}} + 1\right)$$

```
In [12]: diff(phi, z).subs(z, z_pol)
```

Out[12]:

$$V \left(- \frac{R^{2}}{r^{2}} e^{- 2 i \theta} + 1\right)$$

```
In [13]: w = (diff(phi, z)*exp(I*theta)).subs(z, z_pol).subs(exp(I*theta), cos(theta) + I*sin(theta))

In [14]: u_r = re(w).simplify()

In [15]: u_r
```

Out[15]:

$$\frac{V}{r^{2}} \left(- R^{2} + r^{2}\right) \cos{\left (\theta \right)}$$

```
In [16]: u_theta = -im(w).simplify()
```

```
In [17]: u_theta
```

Out[17]:

$$-\frac{V}{r^2}\left(R^2 + r^2\right)\sin\left(\theta\right)$$

```
In [18]: u_theta.subs(r, R)
```

Out[18]:

$$-2V\sin\left(\theta\right)$$

```
In [19]: dUedsTheta = 1/R*diff(u_theta.subs(r, R), theta)

In [20]: dUedsTheta
```

Out[20]:

$$-\frac{2V}{R}\cos\left(\theta\right)$$

```
In [21]: s = Symbol("s", real=True, positive=True)

In [22]: Ue = Function("Ue")(s)

In [23]: Ue = u_theta.subs(r, R).subs(theta, s/R-pi)

In [24]: Ue
```

Out[24]:

$$2V\sin\left(\frac{s}{R}\right)$$

```
In [25]: plot(Ue.subs(V, 1).subs(R, 1), (s, 0, pi))

Out[25]: <sympy.plotting.plot.Plot at 0x7f053f069630>

In [26]: dUeds = dUedsTheta.subs(theta, s/R-pi)

In [27]: dUeds
```

Out[27]:

$$\frac{2V}{R}\cos\left(\frac{s}{R}\right)$$

```
In [28]: plot(dUeds.subs(V, 1).subs(R,1), (s, 0, pi*1.0))

Out[28]: <sympy.plotting.plot.Plot at 0x7f05381fc2e8>

In [29]: nu = Symbol("nu", real=True)

In [30]: sp = Symbol("sp", real=True)

In [31]: integrate((Ue**5).subs(s, sp), (sp, 0, s))
```

Out[31]:

$$32RV^5\left(-\frac{1}{5}\cos^5\left(\frac{s}{R}\right) + \frac{2}{3}\cos^3\left(\frac{s}{R}\right) - \cos\left(\frac{s}{R}\right)\right) + \frac{256R}{15}V^5$$

```
In [32]: theta_t = sqrt(0.45*nu/Ue**6*integrate((Ue**5).subs(s, sp), (sp, 0, s)))

In [33]: theta_t
```

Out[33]:

$$\frac{0.0838525491562421\left|\sin\left(\frac{s}{R}\right)\right|}{V^{3}\sin^{2}\left(\frac{s}{R}\right)}\sqrt{\nu\left(32RV^{5}\left(-\frac{1}{5}\cos^{5}\left(\frac{s}{R}\right)+\frac{2}{3}\cos^{3}\left(\frac{s}{R}\right)-\cos\left(\frac{s}{R}\right)\right)+\frac{256R}{15}V^{5}\right)}$$

```
In [34]: Lambda = theta_t**2/nu*dUeds

In [35]: Lambda
```

Out[35]:

$$\frac{0.0140625\cos\left(\frac{s}{R}\right)}{RV^{5}\sin^{2}\left(\frac{s}{R}\right)}\left(32RV^{5}\left(-\frac{1}{5}\cos^{5}\left(\frac{s}{R}\right)+\frac{2}{3}\cos^{3}\left(\frac{s}{R}\right)-\cos\left(\frac{s}{R}\right)\right)+\frac{256R}{15}V^{5}\right)$$

```
In [36]: L, rho = symbols("L rho", real=True)

In [37]: def S(L):
             return 0.22 + 1.57*L - 1.8*L**2

In [38]: Cf = 2*nu/(Ue*theta_t)*S(Lambda)

In [39]: Cf.simplify()
```

Out[39]:

$$\frac{1.49071198499986\sqrt{30}\nu}{\sqrt{R}\sqrt{V}\sqrt{\nu\left(\cos\left(\frac{s}{R}\right)-1\right)^{3}\left(3\sin^{2}\left(\frac{s}{R}\right)-9\cos\left(\frac{s}{R}\right)-11\right)}\sin^{3}\left(\frac{s}{R}\right)\left|\sin\left(\frac{s}{R}\right)\right|}\left(-0.00162\left(\cos\left(\frac{s}{R}\right)-1\right)^{6}\left(-3\sin^{2}\left(\frac{s}{R}\right)+9\cos\left(\right.\right.\right.$$

```
In [40]: Cf.subs(nu, 1.335).subs(R, 1/24).expand().subs(V, 5).simplify()
```

Out[40]:

$$\frac{1}{\left(-\left(\cos\left(24.0s\right)-1\right)^{3}\left(15.4513888888889\cos\left(24.0s\right)+2.57523148148148\cos\left(48.0s\right)+16.309799382716\right)\right)^{\frac{3}{2}}\sin^{3}\left(24.0s\right)\left|\sin\left(24\right.\right.}$$

```
In [41]: plot(Cf.subs(nu, 1.335).subs(R, 1/24.).subs(V, 5.), (s, 0, 0.08), ylim=(0, 1000), ylabel="$C_f

In [41]:
```

Chapter 2 Problem 16

In[18]:=
```
ClearAll["Global`*"]
```

In[19]:=
```
u[y_] := Ue * (3 / 2 * y / δ - 1 / 2 * (y / δ) ^ 3)
```

In[20]:=
```
θ = Integrate[(1 - u[y] / Ue) * u[y] / Ue, {y, 0, δ}]
```

Out[20]= $\frac{39\,\delta}{280}$

In[21]:=
```
RHS = ν * (D[u[y], y] /. y → 0) / Ue^2
```

Out[21]= $\frac{3\,\nu}{2\,\mathrm{Ue}\,\delta}$

In[22]:=
```
solns = DSolve[{39 / 280 * δ'[x] == 3 ν / (2 * Ue * δ[x]), δ[0] == 0}, δ[x], x]
```

Out[22]= $\left\{\left\{\delta[x] \to -\frac{2\sqrt{\frac{70}{13}}\,\sqrt{x\,\nu}}{\sqrt{\mathrm{Ue}}}\right\}, \left\{\delta[x] \to \frac{2\sqrt{\frac{70}{13}}\,\sqrt{x\,\nu}}{\sqrt{\mathrm{Ue}}}\right\}\right\}$

In[23]:=
```
δ[x_] := Sqrt[280 / 13 * ν * x / Ue]
```

In[24]:=
```
Pr = μ * cp / k
```

Out[24]= $\frac{\mu\,c_p}{k}$

In[25]:=
```
δT[x_] := 1 / (1.026 * Pr^(1 / 3)) * δ[x]
```

In[29]:=
```
T[x_, y_] := (3 / 2 * y / δT[x] - 1 / 2 * (y / δT[x]) ^3) (Te - Tw) + Tw
```

In[31]:=
```
T[x, y] // Simplify
```

Out[31]= $\mathrm{Tw} - \frac{1}{k\,\mathrm{Ue}\left(\frac{x\,\nu}{\mathrm{Ue}}\right)^{3/2}} 0.00540244\,(\mathrm{Te} - 1.\,\mathrm{Tw})\left(1.\,\mathrm{Ue}\,y^3\,\mu\,c_p - 61.382\,k\,x\,y\,\nu\left(\frac{\mu\,c_p}{k}\right)^{1/3}\right)$

In[33]:=
```
qw = -k * D[T[x, y], y] /. y → 0 // FullSimplify
```

Out[33]= $-\frac{0.331613\,k\,(\mathrm{Te} - 1.\,\mathrm{Tw})\left(\frac{\mu\,c_p}{k}\right)^{1/3}}{\sqrt{\frac{x\,\nu}{\mathrm{Ue}}}}$

In[43]:=
```
D[T[x, y], y] /. y → 0 // FullSimplify
```

Out[43]= $\frac{0.331613\,(\mathrm{Te} - 1.\,\mathrm{Tw})\left(\frac{\mu\,c_p}{k}\right)^{1/3}}{\sqrt{\frac{x\,\nu}{\mathrm{Ue}}}}$

In[34]:=
```
h[x_] := -k / (Tw - Te) * D[T[x, y], y] /. y → 0
```

In[36]:= `h[x] // FullSimplify`

Out[36]= $$\frac{0.331613\,k\,\left(\frac{\mu\,c_p}{k}\right)^{1/3}}{\sqrt{\frac{x\,\nu}{Ue}}}$$

In[39]:= `havg = 1 / (L) * Integrate[h[xp], {xp, 0, L}] // FullSimplify`

Out[39]= $$\frac{0.663226\,k\,\left(\frac{\mu\,c_p}{k}\right)^{1/3}}{\sqrt{\frac{L\,\nu}{Ue}}}$$

In[45]:= `Sqrt[(h[x] / havg) ^2 // FullSimplify]`

Out[45]= $$0.5\,\sqrt{\frac{L}{x}}$$

Chapter 2 Problem 18

In[114]:=
```
ClearAll["Global`*"]
```

In[115]:=
```
h[x_, ξ_] := 0.332 * Pr^(1 / 3) * k / x * (ρ * Ue * x / μ) ^ (1 / 2) * (1 - (ξ / x) ^ (3 / 4)) ^ (-1 / 3)
```

In[116]:=
```
xi = {2 * 10 ^ (-2), 5 * 10 ^ (-2)}
```

Out[116]= $\left\{\frac{1}{50}, \frac{1}{20}\right\}$

In[117]:=
```
DT = {100.0, 50.0}
```

Out[117]= {100., 50.}

In[118]:=
```
qw = Sum[h[x, ξ[i]] * ΔT[i], {i, 1, 2}] // Simplify
```

Out[118]= $$\frac{k\,Pr^{1/3}\sqrt{\frac{Ue\,x\,\rho}{\mu}}\left(\frac{0.332\,\Delta T[1]}{\left(1-\left(\frac{\xi[1]}{x}\right)^{3/4}\right)^{1/3}}+\frac{0.332\,\Delta T[2]}{\left(1-\left(\frac{\xi[2]}{x}\right)^{3/4}\right)^{1/3}}\right)}{x}$$

In[119]:=
```
Pr = μ * cp / ρ
```

Out[119]= $\frac{cp\,\mu}{\rho}$

In[120]:=
```
qw = Sum[h[x, xi[[i]]] * DT[[i]], {i, 1, 2}] // Simplify
```

Out[120]= $$\frac{k\left(\frac{45.0593}{\left(20-\sqrt{2}\,5^{1/4}\left(\frac{1}{x}\right)^{3/4}\right)^{1/3}}+\frac{122.31}{\left(50-2^{1/4}\sqrt{5}\left(\frac{1}{x}\right)^{3/4}\right)^{1/3}}\right)\left(\frac{cp\,\mu}{\rho}\right)^{1/3}\sqrt{\frac{Ue\,x\,\rho}{\mu}}}{x}$$

In[121]:=
```
k = 629 * 10 ^ (-6) * 4.184 / 100;
```

In[122]:=
```
R = 4.12 * 10 ^ 3;
```

In[123]:=
```
p = 15 * 101.325 * 10 ^ 3;
```

In[124]:=
```
Ue = 2;
```

In[125]:=
```
γ = 1.405;
```

In[126]:=
```
cp = γ * R / (γ - 1);
```

In[127]:=
```
μ = 126 * 10 ^ (-6) / 10.;
```

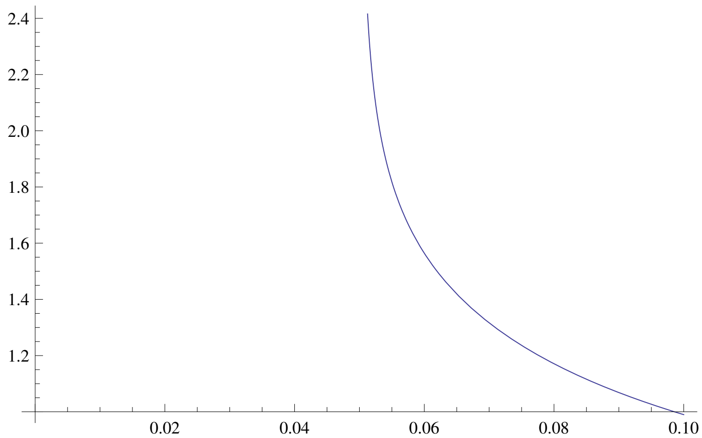
In[128]:=
```
ρ = p / (R * (500 + 273));
```

In[129]:=
```
qw /. x → 6 / 100
```

Out[129]= 1.56426

In[133]:=
```
Plot[qw, {x, 0, 0.1}]
```

Out[133]=



Code listing

Listing 1: Problem 2.1 code

```
#!/usr/bin/env python

import os
import sys
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import rc

#=================================================================
# Setting defaults
#=================================================================

# Setting some defaults
rc("font", **{"family": "serif", "serif": ["Computer Modern"]})
rc("font", size=13.0)
rc("text", usetex=True)
lwidth = 1.5

#=================================================================
# Inputs
#=================================================================

delta = 1.0                         # L
y = np.linspace(0.0, delta, 1000)  # L
Ue = 10.0                           # L/T
u = Ue*np.tanh(2.65*y/delta)       # L/T
rho = 1.0                           # M/L^3
mu = 1.0                            # M/(L*T)

#=================================================================
# Plotting
#=================================================================

# Plotting velocity profile
fig1 = plt.figure(figsize=(3.5,4.5))
plt.plot(u, y, "-k", linewidth=lwidth)
plt.ylabel(r"$y/\delta$")
plt.xlabel(r"$u(y)/U_e$")
plt.xlim([0.0, np.amax(u)+0.1*Ue])
#plt.gca().xaxis.set_ticks([0.0, Ue/2.0, Ue])
plt.gca().xaxis.set_ticks([0.0, Ue])
#plt.gca().set_xticklabels(["0", "$U_e/2$", "$U_e$"])
plt.gca().set_xticklabels(["0", "$U_e$"])
#plt.gca().yaxis.set_ticks([0.0, delta/2.0, delta])
plt.gca().yaxis.set_ticks([0.0, delta])
#plt.gca().set_yticklabels(["0", "$\delta/2$", "$\delta$"])
plt.gca().set_yticklabels(["0", "$\delta$"])
plt.tight_layout()

# Defining delta(x) and Cf(x)
x = np.linspace(0.0001, 1.0, 1000)
Re_x = rho*Ue*x/mu
delta = 5.3242*x*Re_x**(-1.0/2.0)
Cf = 0.9955*Re_x**(-1.0/2.0)
```

```
# Plotting delta(x) and Cf(x)
fig2 = plt.figure(figsize=(8, 5))
ax2 = plt.subplot(2, 1, 2)
plt.plot(x, Cf, "-k", linewidth=lwidth)
plt.xlabel(r"$x$")
plt.ylabel(r"$C_f(x)$")
plt.ylim([0.0, 5.0])
plt.gca().set_yticklabels([])
plt.subplot(2, 1, 1, sharex=ax2)
plt.plot(x, delta, "-k", linewidth=lwidth)
plt.ylabel(r"$\delta(x)$")
plt.gca().set_yticklabels([])
plt.setp(plt.gca().get_xticklabels(), visible=False)
plt.tight_layout()

# Making sure images directory exists
if not os.path.isdir("../Images"):
    os.mkdir("../Images")

# Saving figure
fig1.savefig("../Images/Ch2Prob1a.pdf")
fig2.savefig("../Images/Ch2Prob1b.pdf")

# Showing figure
plt.show()
```

Listing 2: Problem 2.3 code

```
#!/usr/bin/env python

import os
import sys
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import rc

#================================================================
# Setting defaults
#================================================================

# Setting some defaults
rc("font", **{"family": "serif", "serif": ["Computer Modern"]})
rc("font", size=13.0)
rc("text", usetex=True)
lwidth = 1.5

#================================================================
# Inputs
#================================================================

L = 5.0                            # L
x = np.linspace(0.0, L, 1000)      # L
Ue = 1.0/(L-x)                     # L/T
rho = 1.0                          # M/L^3
mu = 1.0                           # M/(L*T)

#================================================================
# Plotting
```

```
#===============================================================

# Plotting momentum thickness
theta = 0.67082*np.sqrt((L-x)*x*mu/rho)
fig1 = plt.figure(figsize=(7, 3.4))
plt.plot(x, theta, "-k", linewidth=lwidth)
plt.xlim([-0.2,L+0.2])
plt.gca().xaxis.set_ticks([0.0, L])
plt.gca().set_xticklabels(["0", "$L$"])
plt.gca().set_yticklabels([])
plt.xlabel(r"$x$")
plt.ylabel(r"$\theta(x)$")
plt.tight_layout()

# Making sure images directory exists
if not os.path.isdir("../Images"):
    os.mkdir("../Images")

# Saving figure
fig1.savefig("../Images/Ch2Prob3.pdf")

# Showing figure
plt.show()
```